COMPOSITE & INVERSE FUNCTIONS

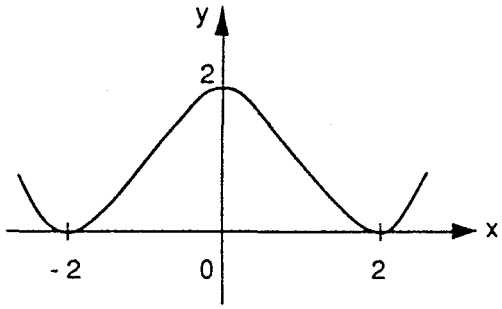
1 $f(x) = 3x^2$ and $g(x) = 2x + 1$.

Work out expressions for the composite functions $f(g(x))$ and $g(f(x))$.

2 For which real values of x are the following functions defined?

$p(x) = \dfrac{1}{2x-6}$ and $q(x) = \sqrt{x^2 - 9}$.

3 *a)* The graph of a function $y = h(x)$ for $-3 \leq x \leq 3$ is shown.



On separate sketches, show the graphs of:

(i) $y = h(x) - 2$, and

(ii) $y = -h(x)$.

b) Assuming the graph has no more turns, what type of function do you think $h(x)$ is?

4 $f(x) = \dfrac{2}{x-1}$

a) Find a formula for $f^{-1}(x)$, the inverses of f and find $f(-1)$ and $f^{-1}(-1)$.

b) Try to find another number with the same property that –1 has in *a*).

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SOLUTIONS

1. $f(x) = 3x^2 \qquad g(x) = 2x + 1$

$f(g(x)) = f(2x+1)$

$= \underline{3(2x+1)^2}$ (2)

$g(f(x)) = g(3x^2)$

$= 2(3x^2) + 1$

$= \underline{6x^2 + 1}$ (2)

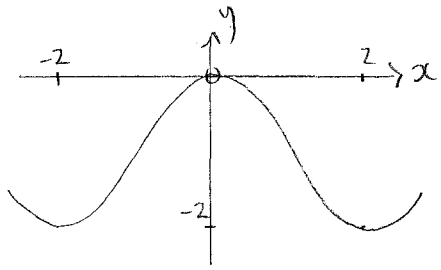
4

2. $p(x) = \dfrac{1}{2x-6}, \quad x \neq 3$ (1)

$q(x) = \sqrt{x^2-9}, \quad x \leqslant -3$ and $x \geqslant 3$ (2)

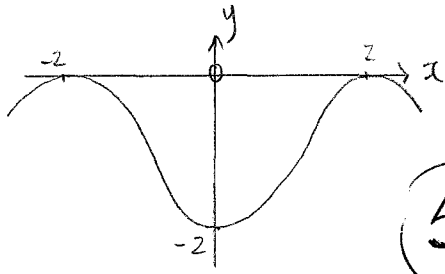
3

3. a) (i)



$y = h(x) - 2$ (2)

(ii)



$y = -h(x)$ (2)

5

b) $h(x)$ is a QUARTIC function. (1)

4 a) $f(x) = \dfrac{2}{x-1} \Rightarrow \underline{f^{-1}(x) = \dfrac{2}{x} + 1}$ (1)

$y = \dfrac{2}{x-1}$ (1)

$y(x-1) = 2$

$x - 1 = \dfrac{2}{y}$ (1)

$x = \dfrac{2}{y} + 1$ (1)

b) $f(-1) = \dfrac{2}{(-1)-1}$ (1)

$= \underline{-1}$

$f^{-1}(-1) = \dfrac{2}{(-1)} + 1$

$= \underline{-1}$ (1)

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HINT: Graph both & check points of intersection

(2)

$\underline{f(2) = f^{-1}(2) = 2}$